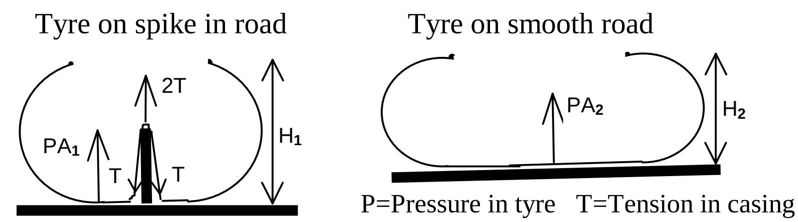
# Variation of Calibration Constant with Surface Texture, Part 3: Modelling of the Deformation Pneumatic Tyres on Different Surfaces

By M.C.W.Sandford, 22 Stevenson Dr., Abingdon, OX14 1SN, UK. Email: m.sandford@lineone.net. 17 Aug 98.

### *Introduction*

In part 1 of this series, ***MN*** 89 p 12, I reviewed the published data on the sensitivity of tyres to the surface texture. In part 2, ***MN*** 90 p 5, I described experimental results from seven riders and twelve tyres on a 4.5 km course in Abingdon. These data showed that for solid tyres the calibration constant in counts/km increases with increasing road roughness, while most pneumatic tyres have a smaller constant on rougher surfaces.

I offered an explanation based on the idea that the solid tyres closely follow the surface contour, whereas in the pneumatic tyre a membrane (the tyre casing wall), stretched by the pressure from the inner tube, carries a tension which resists the deformation by a pointed stone protruding from the road surface. I offered an intuitive argument that by locally wrapping the tyre round protruding points sufficient upward force might be generated to support part of the weight of the front wheel thus requiring a reduction in the general deformation of the tyre. This would increase the effective rolling radius and thus decrease the calibration constant. What I had in mind is illustrated here.




$PA_1+2T=PA_2$ ∴ A=Area of contact patch, $A_2 > A_1$ & $H_1 > H_2$

The article has prompted some correspondence with Bob Letson. (See ***MN*** 87 p 3 for a profile of Bob, who performed studies of surface sensitivity 20 years ago for the AAU's standards committee and discovered rules of thumb which we still use today.) Bob was interested in the effect of riders' weights on surface sensitivity. My reply must have seemed superficial since, while warmly encouraging me to continue obtaining *scientific evidence,* he gently chided me for giving an *intuitive* response to his queries.

Initially I smarted slightly under such admonishment. After all, I have ridden up and down calibration courses in Abingdon several thousand times in the last three years in the search for scientific understanding of the measurement process. One way to satisfy Bob would be to do more riding. However, I decided to discontinue for the moment the tedious collection of data. I have plenty. Where progress must be made is in modelling. By modelling, I mean calculation of the effects.

Modelling has been made much more accessible by the availability of cheap spreadsheet programmes on today's powerful yet cheap PCs. My 150 MHz PC with MS Excel 5 is fast enough to obtain results in seconds which would be statistically revealed only after many hours of riding. The negative aspect was that I had to concentrate for two days on understanding the problem and setting up the model, and a further three days to write this article.

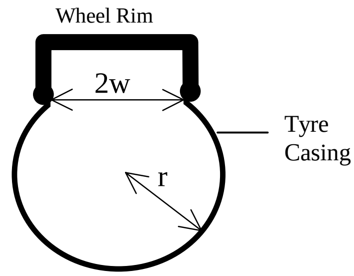
I had two tyre types to model solid and pneumatic. I envisaged working out the effective rolling radius on smooth and rough surfaces and finally trying the effect of changing the rider's weight. I naively thought that modelling the solid tyre might be the easiest since the finite element programmes are used to great effect by engineers for determining the distortions of complex shaped objects. However, reference to text books soon showed that it would be a daunting prospect to understand finite element modelling well enough to set it up anew on my spreadsheet. The principal problem was that I did not immediately see how to use Excel to organise and solve the large number of simultaneous equations needed for the finite element method. Someone with access to commercial finite element software might solve the problem rather easily.

I turned then to the pneumatic tyre. I remembered the seminar in 1991 for beginners when I learnt about measurement. I had then made a foolhardy claim that it would be interesting to model the deformation of a pneumatic tyre in order to understand temperature effects. In fact, the problem defeated me because of the complex geometry. I got the necessary information on temperature effects experimentally by riding my bike up and down calibration courses many times at different temperatures. Now I have succeeded in overcoming the geometric complexity with the aid of Excel. In particular Excel has a tool called SOLVER that enables one to find numerical solutions for transcendental equations such as x=sin(x). In this article, I report the results of modelling the deformation of a pneumatic tyre.

In summary what I have done is use my computer to balance a pneumatic tyre with a range of front wheel weights first on a perfectly smooth road, and then along an extreme surface, a knife edge running along the direction of riding. In each case the deformation of the tyre and the size of the contact patch has been calculated.

I am very surprised by the results which show that my explanation in the second paragraph is probably completely wrong: rolling radius is to the first order independent of the extreme variation in surface form which I modeled. What is even more interesting and important is that the results have focused me sharply on the inadequacies of my intuitive thinking about the role of what I call the effective rolling radius of tyres. I have not yet solved the problem of surface sensitivity but I have obtained new insights which may ultimately provide solutions to this and other problems.

After that grand claim, I am not going to ask the non-technical measurer to read all of the following. Just look at the pictures and then skip to the conclusions. The technical information is provided to enable someone interested to follow the details of my work and check its validity.




### *Model of a Pneumatic Tyre not in Contact with the Road*

This simple model is given by the equations:

$$P = \frac{T}{r}, \qquad T = \frac{E(l-l_0)}{l_0}, \qquad l = 2\pi r - 2r\sin^{-1}\left(\frac{w}{r}\right)$$

$P$ is the pressure in the inner tube = 0.7 N/mm$^2$ (104 psi), $2w$ is the rim width = 30 mm, $l_0$ is the unstretched length of the casing (in cross section direction) = 120 mm., $E$ is the elastic constant of the casing = 100 N/mm (nb this is the modulus of elasticity times the effective casing thickness. I have adjusted E to get a value for the stretching of the tyre casing which is typical for a pneumatic tyre with at pressure of 100 psi). From these values we can deduce the stretched casing length $l$ = 143 mm, the tension in unit width of the casing $T$ = 19.5 N/mm, and the radius of the stretched casing $r$ = 27.9 mm.

### *Assumptions in Model*

I list here the assumptions in the above model:

1. The tyre casing is a thin elastic membrane
2. The tyre casing is anchored at the rim of the wheel by a wire located in the bead and is free to leave the rim at any angle
3. In the radial direction the tyre takes up circular cross-section. I can prove that such a circular cross-section is a shape of stable equilibrium. I have not proved it is the only possible stable shape, but I have never seen another shape in practice.
4. For a wheel radius of 300 mm, the tension in the circumferential direction is ignored since I have calculated that the strain of 0.02 in the circumference when the length of the cross-section of the casing is stretched from 120 mm to 143 mm. This gives a circumferential tension approximately 2 N/mm. A pressure of 0.002 N/mm$^2$ is sufficient to provided this tension. Thus the internal pressure almost wholly supports the radial tension and the effect on the pressure of the circumferential tension can be ignored.
5. Possible coupling between the radial and the circumferential strain through the cross-ply nylon reinforcing of the casing which runs at approximately 45° between the radial and circumferential direction has been ignored. This could significantly affect the assumption that one can independently model the behaviour in the radial and circumferential directions with coupling only through the overall tyre geometry and the internal pressure. I do not have sufficient information about the behaviour of the reinforced casing or the properties of the nylon and rubber to attempt an exact model.

### *Model of a Pneumatic Tyre Deformed on a Smooth Road*

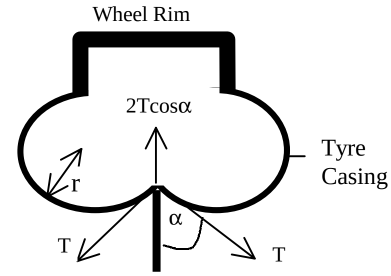
The model of the tyre in contact with a smooth road is obtained by slicing the cross-section of the tyre at intervals of 3 mm. The tyre pressure can then be used to calculate the tyre tension and radius in a similar fashion to that above. The difference is that a portion of the tyre is in contact with the ground. This portion lies flat along the ground and the internal pressure causes a force on the ground. Integrated over the whole area of the contact patch this provides an upward force which will be equal to the weight of the wheel on the ground. The calculation has been done by setting the equations up in a spreadsheet, reproduced on the following page. The total wheel force of 212.9 N (48 lbs) is derived in cell L23. The undeformed tyre has a thickness of 51.3 mm. Immediately beneath the axle at theta = 0, where the thickness is 47.0 mm, the tyre has been squashed by 4.3 mm.

| | A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | **CONSTANTS** | | | | Elastic const = | 100 | rim half width = | 15 | | | | |
| 4 | Rim radius R = | 300 | | | Pressure = | 0.7 | | | | | | |
| 5 | Thickness x = | 47 | | | Tyre casing = | 60 | | | | | | |
| 6 | Theta, angle from vertical in radians | (R+x)/cos(theta)-R | (R+x)tan(theta) | | tyre radius, r | l, casing length | Difference in l | l, casing length | | Contact Patch width | Force=Area*pressure | Cumulative force |
| 7 | | '=($B$4+$B$5)/COS(A8)-$B$4 | '=($B$4+$B$5)*TAN(A8) | | | '=$F$5*(1+$F$4*E8/$F$3) | '=(F8-H8)^2 | '=$H$3-SQRT(2*B8*E8-B8*B8)+PI()*E8-E8*IF(B8<2*E8,ACOS((B8-E8)/E8),10000) | '=ACOS(B8/E8-1) | '=$H$3-SQRT(2*B8*E8-B8*B8) | '=$F$4*2*J8*(A9-A8)*($B$5+$B$4) | '=SUM(K$8:K8) |
| 8 | 0.00000 | 47.00000 | 0.00000 | | 24.79734 | 70.41488 | 9.58198E-10 | 70.41491 | 0.46155 | 3.95691 | 19.22267 | 19.22267 |
| 9 | 0.01000 | 47.01735 | 3.47012 | | 24.80916 | 70.41985 | 1.42005E-09 | 70.41981 | 0.46200 | 3.94148 | 19.14769 | 38.37037 |
| 10 | 0.02000 | 47.06941 | 6.94093 | | 24.84466 | 70.43476 | 4.76653E-08 | 70.43454 | 0.46338 | 3.89516 | 18.92271 | 57.29307 |
| 11 | 0.03000 | 47.15621 | 10.41312 | | 24.90387 | 70.45963 | 8.37131E-08 | 70.45934 | 0.46565 | 3.81804 | 18.54804 | 75.84111 |
| 12 | 0.04000 | 47.27779 | 13.88741 | | 24.98690 | 70.49450 | 1.34633E-09 | 70.49446 | 0.46882 | 3.71011 | 18.02373 | 93.86484 |
| 13 | 0.05000 | 47.43420 | 17.36447 | | 25.09397 | 70.53947 | 8.78728E-08 | 70.53976 | 0.47287 | 3.57102 | 17.34803 | 111.21287 |
| 14 | 0.06000 | 47.62554 | 20.84502 | | 25.22533 | 70.59464 | 4.90513E-07 | 70.59534 | 0.47781 | 3.40052 | 16.51971 | 127.73259 |
| 15 | 0.07000 | 47.85189 | 24.32975 | | 25.38127 | 70.66013 | 1.40212E-06 | 70.66132 | 0.48361 | 3.19830 | 15.53734 | 143.26993 |
| 16 | 0.08000 | 48.11337 | 27.81937 | | 25.56233 | 70.73618 | 3.83909E-07 | 70.73680 | 0.49029 | 2.96324 | 14.39540 | 157.66533 |
| 17 | 0.09000 | 48.41011 | 31.31460 | | 25.76876 | 70.82288 | 1.40019E-10 | 70.82287 | 0.49780 | 2.69559 | 13.09518 | 170.76051 |
| 18 | 0.10000 | 48.74226 | 34.81613 | | 26.00078 | 70.92033 | 3.29577E-07 | 70.92090 | 0.50609 | 2.39581 | 11.63883 | 182.39934 |
| 19 | 0.11000 | 49.10999 | 38.32470 | | 26.25894 | 71.02876 | 4.17605E-06 | 71.03080 | 0.51515 | 2.06316 | 10.02282 | 192.42216 |
| 20 | 0.12000 | 49.51348 | 41.84103 | | 26.54466 | 71.14876 | 1.48893E-08 | 71.14864 | 0.52507 | 1.69389 | 8.22893 | 200.65109 |
| 21 | 0.13000 | 49.95294 | 45.36585 | | 26.85748 | 71.28014 | 2.56341E-08 | 71.27998 | 0.53567 | 1.29148 | 6.27399 | 206.92509 |
| 22 | 0.14000 | 50.42859 | 48.89990 | | 27.19833 | 71.42330 | 1.40526E-07 | 71.42368 | 0.54697 | 0.85417 | 4.14958 | 211.07467 |
| 23 | 0.15000 | 50.94069 | 52.44392 | | 27.56809 | 71.57860 | 6.17991E-07 | 71.57938 | 0.55895 | 0.38080 | 1.84994 | 212.92460 |
| 24 | 0.16000 | 51.48948 | 55.99867 | | 27.96808 | 71.74659 | 2.5437E-06 | 71.74500 | | -0.13132 | NOT CONTACTING GROUND | |
| 25 | | | | | | Sum (g8:g24)= | 1.0348E-05 | | | | | |

***SPREADSHEET CALCULATION OF DEFORMATION OF PNEUMATIC TYRE ON SMOOTH ROAD***

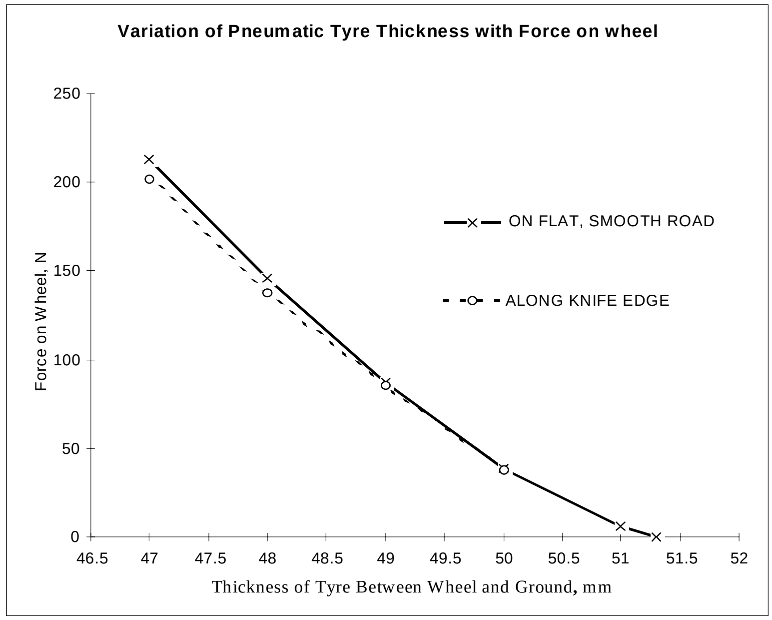
Row 7 shows the spreadsheet formulae which underlie row 8 onwards. A series of angles is entered in column A, and approximate solutions for the radius of the casing are entered in column E. The stretched length of the casing, l, is calculated by two different methods. In column F the tyre pressure is used to derive the casing stress and the length is then calculated using the coefficient of elasticity. In column H the length is calculated from geometrical considerations. The square of the difference of columns F and H appears in column G. To solve the model, the Excel ***SOLVER*** tool is used to minimise the value of cell G25, the sum of column G, by varying the cells in column E, the radius. The calculation is then repeated for different values of the deformed thickness in cell B5. The units are N and mm.

## *Model of a Pneumatic Tyre on a Knife Edge in the Road*




In the calculation for the tyre balanced on a knife the contact patch is a narrow line. The upward force is provided indirectly by the pressure in the tyre. The tension, T, in the casing draped over the knife edge causes the upward reaction which supports the wheel.

As above, I assumed a number of values for the distance between the wheel rim and the knife edge, and for each value calculated the length of the contact strip and the total upward force. I found that for small deformations of the tyre, the upward force was similar in both models. But for large deformations with upward forces over 100N, the flat surface could provide the upward force for a slightly smaller value of tyre deformation than when on the knife edge. This was the opposite to what I would expect if sharp stones in the road were having the effect of increasing the thickness of the tyre, and hence increasing the effective rolling radius.




To illustrate the magnitude of the discrepancy with experimental results, my Michelin Tracer pneumatic tyre gives about 0.05% larger effective rolling radius on a rough road with about 250N weight on the wheel. The graph of the results of the modelling, shown here, predicts that the radius will be about 0.2 mm or 0.06 % *smaller* when on the knife edge.

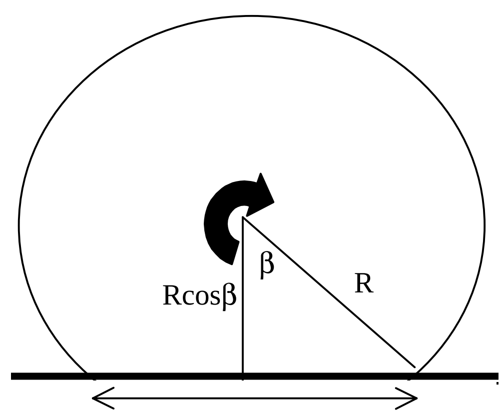
## *Assumptions in Calculating the Interaction with the Road*

I listed above the assumptions I made in my basic model of the tyre not in contact with the road. Here are the additional assumptions which I think have made in the models of interaction with the road. Incorrect assumptions could be the cause of the model failing to give the expected result. Alternatively relevant assumptions may have been overlooked.

1. The tyre meets the flat road tangentially in the direction of the cross-section.
2. The stresses and strains in the circumferential direction have negligible coupling to the cross-section direction, so that the an accurate model can be made by taking cross-sections of the tyre shape.
3. In interpreting the consequences on the calibration constant for measurement purposes, I have made the assumption that the axle to ground distance defines the effective rolling radius. I examine this important assumption in the next section.

## *A Deformed Rolling Tyre: Effective Radius*

Implicit in the discussion so far has been the assumption that the effective rolling radius is given by the distance between the axle and the ground. If there is no slipping between the tyre and the ground and if the axle location remains at the centre of the wheel, then some simple geometry leads me to the conclusion that the assumption is true.




If $R_e$ is the effective rolling radius, then as the bike moves forward a small distance $\delta x$, the angle through which the wheel turns is $\delta x/R_e$. Imagine now the small length, $R^*\delta x/R_e$ of undeformed tyre at the end of the radius in the adjacent figure, just about to contact the ground, as the wheel moves forward $\delta x$. If this pi n it touches the ground, then it must approach the ground with no horizontal component of velocity. This statement would not necessarily be true for a light tyre with no mass which needed acceleration. I assume that the tyre is a heavy membrane having a finite mass per unit area. Thus any area making contact with the ground with a relative horizontal velocity will need an impulse to remove the relative velocity and this will cause at least momentary slipping contrary to my assumption of no slipping. The horizontal component of displacement of the tyre is δx

$$\delta x = \frac{R}{R_e}\delta x \cos(\beta) \text{ which gives the effective rolling radius, } R_e = R\cos(\beta) \text{ which is the axle - ground separation.}$$

towards the right and R*δx/$R_e$ cosβ toward the left. For no relative motion,

Another interesting aspect of the mechanics of the rolling deformed wheel is that the tyre is placed under circumferential compression. The small element R*δx/$R_e$ considered above is compressed to length δx on making contact with the road. This is a compressive strain of R/$R_e$ – 1. The whole length in contact with the road is under an average compressive strain of Rβ/$R_e$sinβ-1 = 2β/sin(2β)-1. The initial strain actually increases as the segment passes beneath the axle. For a typical value of β=0.1, this gives a strain of about 0.7%. Such a small strain gives a negligible additional circumferential force of about 0.7 N per mm of width, which is small compared to the vertical force at the contact point. I conclude that this strain is unlikely to make the tyre slip unless the road is icy. The assumption of no slip is therefore a good one, except possibly very close to the point of first contact.

If it can not be explained by slip then the only other explanations I can suggest for the behaviour of the pneumatic tyre on rough surfaces are

1. The tyre deformation extends beyond the contact point. This may be different for solid and pneumatic tyres.
2. The varying height of the irregularities on the road will certainly modify the assumptions in the above model of the contact point region.

## *Conclusions*

My model takes as inputs: the wheel radius, the rim width, the tyre casing width and the coefficient of elasticity, the inner tube pressure, and the weight on the front wheel. It enables the following to be derived: the distance from the axle to the ground (which is what I had intuitively identified as the rolling radius), the shape and size of the contact patch, the shape of the deformed tyre casing.

The general behaviour on a smooth flat road is broadly consistent with my qualitative observations of pneumatic tyres, so I know I have chosen reasonable values for such unmeasured items as the coefficient of elasticity of the tyre casing. I could make a more detailed verification of the model by comparing the results with detailed measurements of the contact patch on several tyres.

When placed on the knife edge the deformation is very slightly *more* than on the flat surface. This is completely inconsistent with my previous intuitive explanation. I thought that the local deformation of the tyre round a sharp edge would provide an upward force by virtue of the internal tension, which would reduce the amount of weight supported through the general deflection of the tyre. I had guessed that the net result would be to increase the axle to ground separation. The model gives a small reduction in separation, which one would naively expect to give a larger constant. However, the experimental results reported in part 2 show this does not occur with most pneumatic tyres which I have tested. I therefore have to search for another explanation for the pneumatics tyre's surface sensitivity.

I have arrived at a new understanding of the rotating deformed tyre. There are two contributions to the effective rolling radius and hence to the calibration constant. The axle-ground separation is the dominant parameter. In one sense it is determined by the deformation of the tyre as calculated in this article. In another sense it is caused by the circumferential compression of the tyre in contact with the road. There is a circumferential compression as each element of the tyre contacts the road, and there is further compression as it passes under the axle. Without this compression the effective rolling radius would equal the unloaded radius of the tyre. It appears to me that this basic geometrical result is not dependent on the surface roughness.

Surface roughness effects probably arise in the region near the point of first contact between the wheel and the ground where they affect the amount of initial circumferential compression of the tyre. I speculate that there are three possible causes: 1) tyre deformation extending beyond the point of first contact, 2) road height irregularities modifying the geometry of initial contact, 3) varying skidding at the point of first contact.

I think I can now give a more scientifically based answer to Bob Letson's query. Rider weight will change the axle-ground separation, but this is likely to have only a second order effect on the processes at the point of first contact. Therefore, I expect only a small variation of surface sensitivity with rider weight.